

# Getting bounds on the mixing angles for a nonsequential bottom quark

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We analyze the  $Zf\bar{f}$  vertex in the framework of models that add a new bottom quark in a nonsequential way and we evaluate the tree level contribution to the CERN, LEP, or SLC observables  $\Gamma_Z$ ,  $R_l$ , and  $R_b$ . We obtain bounds for the mixing angles from the experimentally allowed contour regions of the parameters  $\Lambda_{L,R}$  introduced here. In order to get a more restrictive region, we consider the experimental results for  $B \rightarrow \nu\bar{\nu}X$  as well.

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## I. INTRODUCTION

The comparison of theoretical predictions with experimental data has confirmed the validity of the standard model (SM) in an impressive way. The quantum effects of the SM have been established at the  $1\sigma$  level, and the direct and indirect determinations of the top quark mass are compatible with each other. In spite of this success, the conceptual situation with the SM is not completely satisfactory for a number of deficiencies. Some of them are the large number of free parameters and the hierarchical fermion masses.

The SM contains three generations of quarks in irreducible representations of the gauge symmetry group  $SU(3)_C \times SU(2)_L \times U(1)_Y$ . The possibility of extending them has been studied in different frameworks [1–8] which are based either on a fourth generation sequential family, or on nonsequential fermions, regularly called exotic representations because they are different from those of the SM.

These unusual representations emerge in other theories, such as the  $E_6$  model where a singlet bottom type quark appears in the fundamental representation [2]; also, top-like singlets have been suggested in supersymmetric gauge theories [3]. The principal feature of a model which extends the quark sector with an exotic fermion is that there are new quark mixing phases in addition to the single phase of the SM. Therefore, in this kind of models  $Z$  boson mediated flavor changing neutral currents (FCNC's) arise at the tree level. This fact can affect the mixing mechanism in the neutral  $B$ -system [1–6].

The possibility of indirect consequences of singlet quark mixing for FCNC and  $CP$  violation has been used to get bounds on the flavor changing couplings. Heavy meson decays like  $B^0$  and  $D^0 \rightarrow \mu^+ \mu^-$  [1,5], rare decays  $b \rightarrow s \gamma$  [1,2,5], measurements such as  $K_L \rightarrow \mu^+ \mu^-$ ,  $B \rightarrow \mu^+ \mu^- X$ ,  $B \rightarrow \nu\bar{\nu}X$ ,  $K$  meson physics [1,2,4], or even  $Z \rightarrow l\bar{l}$ ,  $l \rightarrow l\bar{l}$  [9] have been considered for this purpose.

In the last years, the CERN  $e^+e^-$  collider LEP and SLAC (SLC) Linear Collider have brought to completion a remarkable experimental program by collecting an enormous amount of electroweak precision data on the  $Z$  resonance. This activity, together with the theoretical efforts to provide accurate SM predictions have formed the apparatus of electroweak precision tests [10]. We are interested in using the electroweak precision test quantities in order to get bounds

on the mixing angles for additional fermions in exotic representations. Specifically, we want to consider models that include a new quark with charge  $-1/3$  which is mixed with the SM bottom quark. This kind of new physics was taken into account by Bamert *et al.* [11] during the discrepancy between experiment and SM theory in the  $R_b$  ratio. They analyzed a broad class of models in order to explain the discrepancy, and they considered those models in which new  $Zb\bar{b}$  couplings arise at the tree level through  $Z$  or  $b$  quark mixing with new particles.

Our presentation is based on the parametrization of the  $Zf\bar{f}$  vertex in an independent model formulation. Therefore these results can be used for different quark representations like singlet down quark, vector doublets model, mirror fermions and self conjugated triplets, etc. The parametrization of the vertex in a general way has been reviewed by Barger *et al.* [1,4] as well as Cotti and Zepeda [9]. The LEP precision test parameters that we use are the total  $Z$  width  $\Gamma_Z$ ,  $R_l$ , and  $R_b$ .

The procedure to get bounds on the mixing angles is the following. First, we analyze the  $Zf\bar{f}$  vertex as obtained after a rotation of a general quark multiplet (common charge) into mass eigenstates. In particular, we write down the neutral current terms for the bottom quarks, which are assumed to be mixed. With these expressions we can evaluate the tree level contribution to the process  $Z \rightarrow b\bar{b}$ ; we enclose this new contribution within the coupling constants  $g_V$  (vectorial) and  $g_A$  (axial). We then write down  $\Gamma_Z$ ,  $R_l$ , and  $R_b$  including the new contributions, and we obtain bounds on the new parameters by using the experimental values from LEP and SLC [12]. Finally, we do a  $\chi^2$  analysis and find the allowed region in the plane of the new parameters  $\Lambda_L$  and  $\Lambda_R$  introduced. We also use the result obtained by Grossman *et al.*, involving  $B \rightarrow \nu\bar{\nu}X_s$  [4], in order to narrow down the bounds in the contour plots.

## II. PRECISION TEST PARAMETERS

To restrict new physics, we will use parameters measured at the  $Z$  pole. These parameters are the total decay width of the  $Z$  boson  $\Gamma_Z$ , the fractions  $R_b = \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{hadrons})$  and  $R_l = \Gamma(Z \rightarrow \text{hadrons})/\Gamma(Z \rightarrow l\bar{l})$  [10,12]. Considering the new physics (NP) and the SM couplings, we can write

$$\Gamma(Z \rightarrow b\bar{b}) = \frac{G_F m_Z^3}{6\sqrt{2}\pi} \left[ \frac{3\beta - \beta^3}{2} (g_V^{SM} + g_V^{NP})^2 + \beta^3 (g_A^{SM} + g_A^{NP})^2 \right] N_C R_{QCD+QED}, \quad (1)$$

where  $N_C$  is the number of colors,  $R_{QCD+QED}$  are the QCD and QED corrections, and  $\beta = \sqrt{1 - 4(m_b^2/m_Z^2)}$  is the kinematic factor [10] with  $m_b = 4.7$  GeV. We also are taking into account the oblique and vertex contributions to  $g_{V,A}^{SM}$  giving by the top quark and Higgs boson. For our purpose, It is convenient to separate the SM and NP contributions as follows:

$$\Gamma(Z \rightarrow b\bar{b}) = \Gamma_b^{SM} (1 + \delta_b^{NP}). \quad (2)$$

The symbol  $\delta_b^{NP}$  is given by

$$\delta_b^{NP} = \frac{(3 - \beta^2)[(g_V^{NP})^2 + 2g_V^{NP}g_V^{SM}] + 2\beta^2[(g_A^{NP})^2 + 2g_A^{NP}g_A^{SM}]}{(3 - \beta^2)(g_V^{SM})^2 + 2\beta^2(g_A^{SM})^2}. \quad (3)$$

This equation could be written using the new physics parameters  $\Lambda_{L(R)}$  that were introduced in Eq. (18), through the relationships  $g_A^{NP} = \Lambda_L - \Lambda_R$  and  $g_V^{NP} = \Lambda_R + \Lambda_L$ .

Similarly, the  $Z$  decay into hadrons after considering the NP, can be written as

$$\begin{aligned} \Gamma(Z \rightarrow \text{hadrons}) &= 2\Gamma_u^{SM} + 2\Gamma_d^{SM} + \Gamma_b \\ &= \Gamma_{had}^{SM} \left( 1 + \frac{\Gamma_b^{SM}}{\Gamma_{had}^{SM}} \delta_b^{NP} \right). \end{aligned} \quad (4)$$

Here, only  $\Gamma_b$  gets NP corrections because only the SM bottom mixes with the exotic quark. Therefore, the  $Z$  partial decay into  $d$  and  $s$  quarks remains unchanged.

On the other hand,  $\Gamma_Z$  is equal to

$$\Gamma_Z = 3\Gamma(Z \rightarrow \nu\bar{\nu}) + 3\Gamma(Z \rightarrow l\bar{l}) + \Gamma(Z \rightarrow \text{hadrons}) \quad (5)$$

which again is rewritten, with Eq. (4), as follows:

$$\Gamma_Z = \Gamma_Z^{SM} \left( 1 + \frac{\Gamma_b^{SM}}{\Gamma_Z^{SM}} \delta_b^{NP} \right). \quad (6)$$

Using the above equations, for  $R_l$  and  $R_b$  we obtain the following expressions:

$$\begin{aligned} R_l &= R_l^{SM} (1 + R_b^{SM} \delta_b^{NP}), \\ R_b &= R_b^{SM} [1 + \delta_b^{NP} (1 - R_b^{SM})]. \end{aligned} \quad (7)$$

In a general way,  $R_b$  is mainly a measure of  $|g_L^b|^2 + |g_R^b|^2$ ; therefore, the fraction  $R_b$  is very sensitive to anomalous couplings of the  $b$  quark.

### III. THE MODEL

Following closely the notation of Ref. [9], if we have a multiplet  $\Psi_{a=L,R}^O$  with  $n_a$  ordinary fermions and  $m_a$  exotic fermions with the same electric charge  $q$ :

matic factor [10] with  $m_b = 4.7$  GeV. We also are taking into account the oblique and vertex contributions to  $g_{V,A}^{SM}$  giving by the top quark and Higgs boson. For our purpose, It is convenient to separate the SM and NP contributions as follows:

$$\Psi_a^O = U_a \Psi_a, \quad \Psi_a = \begin{pmatrix} \Psi_l \\ \Psi_h \end{pmatrix}_a, \quad (8)$$

where  $U_a$  is the unitary matrix that rotates the mass eigenstate  $\Psi_a$  into the interaction eigenstate  $\Psi_a^O$ .  $\Psi_{l(h)}$  means ordinary or light (exotic or heavy) fermions.  $U_a$  can be further the composed as follows [9]:

$$U_a = \begin{pmatrix} A & E \\ F & G \end{pmatrix}_a, \quad (9)$$

where

$$(U^+ U)_a = \begin{pmatrix} A^+ A + F^+ F & A^+ E + F^+ G \\ E^+ A + G^+ F & E^+ E + G^+ G \end{pmatrix}_a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad (10)$$

If we suppose that the up quark sector of the SM is diagonal and that there are no exotic quarks, then  $A_L$  corresponds to the classical Kobayashi-Maskawa matrix. In the SM this matrix is unitarity, whereas in our model it is not:

$$(A^+ A)_L = I - (F^+ F)_L. \quad (11)$$

$F_L$  corresponds to the mixing of the ordinary-exotic quarks. As mentioned,  $A_L$  is not quite unitary and the factor  $(F^+ F)_L$  indicates Flavor Changing transitions in the light-light sector.

The neutral current Lagrangian for the multiplet  $\Psi$  is given by

$$\begin{aligned} -\mathcal{L}^{NC} &= \frac{e}{c_w s_w} \sum_{a=L,R} \bar{\Psi}_a^O \gamma^\mu D_a \Psi_a^O Z_\mu^0 \\ &= \frac{e}{c_w s_w} \sum_{a=L,R} \bar{\Psi}_a \gamma^\mu U_a^+ D_a U_a \Psi_a Z_\mu^0, \end{aligned} \quad (12)$$

where  $s_w = \sin \theta_w$  and  $D_a$  are diagonal matrices which contain the couplings of the neutral  $Z^0$  gauge boson to the matter fields; they have the form

$$D_a = (T_3 - Qs_w^2)_a = \begin{pmatrix} t_{30} - qs_w^2 & 0 \\ 0 & t_{3E} - qs_w^2 \end{pmatrix}_a, \quad (13)$$

where  $T_{3a}$  and  $Q$  are the matrices of the isospin charges and the electric charge, respectively.  $t_{30}$  and  $t_{3E}$  are the standard and exotic weak isospin third component of the multiplets. Using the unitarity relations of the  $U_a$  matrix from Eq. (10), the product  $(U^\dagger DU)_{a=L,R}$  in Eq. (12) can be written as

$$(U^\dagger DU)_L = \begin{pmatrix} F^\dagger F & -A^\dagger E \\ -E^\dagger A & -E^\dagger E \end{pmatrix}_L (t_{3E} - t_{30})_L + T_{3L} - Qs_w^2,$$

$$(U^\dagger DU)_R = \begin{pmatrix} F^\dagger F & F^\dagger G \\ G^\dagger F & G^\dagger G \end{pmatrix}_R t_{3ER} - Qs_w^2. \quad (14)$$

The neutral current Lagrangian in the light-light sector can be written as [9]

$$\mathcal{L}^{NC} = -\frac{e}{c_w s_w} \sum_{a=L,R} \bar{\Psi}_{l,a} \gamma^\mu K_a \Psi_{l,a} Z_\mu^0, \quad (15)$$

where

$$K_L = (F^\dagger F)_L (t_{3EL} - t_{30L}) + I_{3 \times 3} (t_{30L} - qs_w^2),$$

$$K_R = (F^\dagger F)_R t_{3ER} - I_{3 \times 3} qs_w^2. \quad (16)$$

For the SM with three generations,  $K_{L,R}$  are  $3 \times 3$  matrices. They can be produced FC transitions at the tree level depending of  $F_{L,R}$  entries which are the mixing angles of the ordinary and exotic fermions.

In this work, we only consider one *bottom* exotic quark (i.e., not mixing with  $d$  and  $s$ ). Then,  $U_a$  and the  $F^\dagger F$  product become

$$U_a = \begin{pmatrix} & 0 \\ A & 0 \\ & -s_a \\ 0 & 0 & s_a & c_a \end{pmatrix}, \quad (F^\dagger F)_a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sin^2 \theta_a \end{pmatrix}, \quad (17)$$

where  $\sin \theta_{L,R}$  represent the mixing between bottom quark with the exotic ones. Therefore, the coupling  $bbZ$  gets modified by the  $\Lambda_{L,R}$  factors:

$$K_L^b = \Lambda_L + t_{30L} - qs_w^2$$

$$= \sin^2 \theta_L \left( t_{3EL} + \frac{1}{2} \right) + \left( -\frac{1}{2} + \frac{1}{3} s_w^2 \right),$$

$$K_R^b = \Lambda_R - qs_w^2$$

$$= \sin^2 \theta_R t_{3ER} + \left( \frac{1}{3} s_w^2 \right). \quad (18)$$

TABLE I. SM predictions and experimental values measured at LEP for the  $\Gamma_Z$ ,  $R_l$ , and  $R_b$ .

	Experimentals	Standard model
$\Gamma_Z$	$2.4939 \pm 0.0024$	2.49582
$R_l$	$20.765 \pm 0.026$	20.7468
$R_b$	$0.21656 \pm 0.00074$	0.215894

#### IV. RESULTS

With the expressions for  $\Gamma_Z$ ,  $R_l$ , and  $R_b$  in terms of the new physics contribution in Sec. III, and with the experimental data from LEP we get bounds on the parameters  $\Lambda_{L,R}$  introduced in Eq. (18). The experimental data that we used for the LEP parameters, as well as their SM values are in Table I [10,12].

We do a  $\chi^2$  fit of the observables  $\Gamma_Z$ ,  $R_l$ , and  $R_b$ , and then we proceed to obtain bounds on the parameters  $\Lambda_{L,R}$  by taking on values in the best region allowed for them at 95% C.L. This region is displayed in Fig. 1. In order to get a more restrictive region we use the bound  $|\Lambda_{L,R}| < 0.0018$  obtained by Grossman *et al.* [4], which is represented by straight lines in the figure. The intersection between the two regions is given by

$$-1.094 \times 10^{-4} \leq \Lambda_L \leq 1.086 \times 10^{-4},$$

$$-1.8 \times 10^{-3} \leq \Lambda_R \leq 1.8 \times 10^{-3}. \quad (19)$$

We note that for  $\Lambda_L$  the region is more restrictive than the one obtained by Grossman *et al.* [4], while the  $\Lambda_R$  parameter is not modified.

If we consider only mixing between an exotic bottom quark with the third SM family, independent of any group representations, it is given by a  $2 \times 2$  unitary matrix for left-

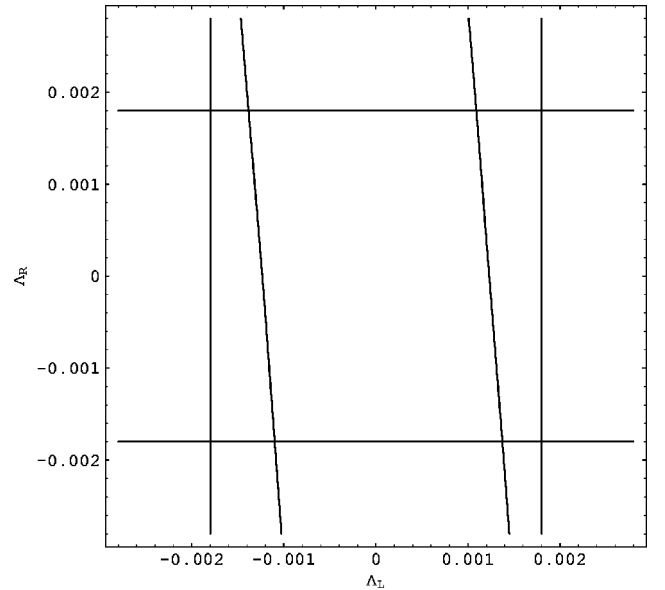


FIG. 1. Contour plot represents the allowed region for  $\Lambda_L$  –  $\Lambda_R$ . The straight lines are the bounds from  $B \rightarrow \nu \bar{\nu} X$  reported in Ref. [4].

TABLE II. The parameters  $\Lambda_{L,R}$  for different representations according with the quantum numbers in Eq. (15).

$(t_{EL}^3, t_{ER}^3)$	$\Lambda_L$	$\Lambda_R$	Model
(0,0)	$\frac{1}{2}\sin^2\theta_L$	0	Vector singlets
$(-\frac{1}{2}, -\frac{1}{2})$	0	$-\frac{1}{2}\sin^2\theta_R$	Vector doublets
$(0, -\frac{1}{2})$	$\frac{1}{2}\sin^2\theta_L$	$-\frac{1}{2}\sin^2\theta_R$	Mirror fermions
$(-1, -1)$	$-\frac{1}{2}\sin^2\theta_L$	$-\sin^2\theta_R$	Self-conjugated triplets

and right-handed fermions. The couplings for several  $SU(2)_L$  representations are given in Table II. We can use these bounds in order to get constraints for the left and right mixing angles of each model. They are shown in Table III.

Summarizing, we have used the fractions  $\Gamma_Z$ ,  $R_l$ , and  $R_b$  to obtain bounds on the mixing angles of new quark bottom-type representations with the SM bottom quark. Taking into account the results of Grossman *et al.* [4], we have gotten the allowed intervals  $-1.094 \times 10^{-4} \leq \Lambda_L \leq 1.086 \times 10^{-4}$  and  $-1.8 \times 10^{-3} \leq \Lambda_R \leq 1.8 \times 10^{-3}$ . Our results reduce the allowed region for the parameter  $\Lambda_L$  while the parameter  $\Lambda_R$  is

TABLE III. Bounds on the mixing angles for different representations of the exotic quarks.

Model	$ \sin\theta_L  \leq$	$ \sin\theta_R  \leq$
Vector singlets	$4.661 \times 10^{-2}$	0
Vector doublets	0	$6 \times 10^{-2}$
Mirror fermion	$4.661 \times 10^{-2}$	$6 \times 10^{-2}$
Self-conjugated triplets	$4.679 \times 10^{-2}$	$4.24 \times 10^{-2}$

not modified with respect to the results obtained by Grossman *et al.* [4]. We may note that the results have been obtained from the tree level contributions, and we can get bounds on the mass of the new quark using oblique corrections [13].

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